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# **Experimental Verification of Solidification Stress Theory**

by

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# Objectives

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- **Crack-free ceramic waste form (CWF)**
- **Analyze Temperature Data to Analyze Cracking in CWF2**
- **Use Sound to Determine when Cracking Occurs**
- **Use Destructive Testing to Determine Where Cracking Occurs**
- **Verify Solidification Stress Theory**
- **Apply Solidification Stress Theory to explain the cracking in CWF2**



# Glass/Ceramic Cylinders Crack During Formation

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- **Cracking (always) occurs in forming large glass or ceramic cylinders (>15 inch diameter)**
- **Cracking occurs in the cooldown from the formation temperature**
- **A crack-free formation is desired**
- **Crack free - more resistance to leaching**



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# Existence of New Stress Proposed

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- **Model developed here proposes a new stress - Solidification Stress**
- **Permanent stress set-in at high temperature - during solidification – Solidification stress**
- **This stress causes failure as the CWF nears room temperature.**
- **Thermal stress does not cause this failure**



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# Test for Solidification Stress

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- **Solidification stress is in addition to and of opposite sign of the thermal stress**
- **Cracking of the CWF would occur at low temperatures if solidification stress exists**
- **Cracking would occur at high temperatures if it does not**
- **Cracking would start in inner region**



# Thermal Stress Model (not pre-stressed)

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- **Cylinder made up of concentric annuli**
- **All intrinsically attached in a solid cylinder**
- **Length determined by average temperature**
- **Each annulus same length as the average**
- **Above  $T_{avg}$  forced shorter - in compression**
- **Below  $T_{avg}$  forced longer so are in tension.**



# Thermal Stress Model (not pre-stressed)

- Not pre-stressed means no thermal stress when all annuli are at the same temperature
- The amount that annulus  $i$  must be elongated to reach the average length,  $L$ , is

$$\Delta L_i = L\alpha(T_{avg} - T_i)$$

where  $\alpha$  = thermal expansion coefficient

$T_i$  = Temperature of annulus  $i$

$T_{avg}$  = Radially averaged temperature



# Thermal Stress Model (not pre-stressed)

- Stress,  $\sigma_i$ , induced into an annulus elongated by a length,  $\Delta L_i$ , is

$$\sigma_i = \frac{E}{1 - \mu} \frac{\Delta L_i}{L}$$

where  $E = \text{Young's Modulus}$

$\mu = \text{Poisson's ratio}$



# Thermal Stress Model (not pre-stressed)

- Combine the two equations to get **Axial Stress at Midplane at any r**

$$\sigma_i = \frac{E \alpha}{1 - \mu} (T_{avg} - T_i)$$

- **Stress at surface largest for cooling cylinder**
- **Circumferential surface stress same as the axial**



# Pre-stresses

- A stress that exists when the temperature is uniform is pre-stress
- Pre-stress induced by solidification is set-in stress
- Total  $\sigma = \text{set-in } \sigma + \text{thermal } \sigma$



# Set-in Stress Model

- **CWF concentric annuli – start all liquid**
- **Surface lowest T; annuli solidify inwards**
- **Each same length when solidifying  $L(T_s)$**
- **When second annulus solidifies, outer is shorter due to thermal contraction.**
- **Two intimately connected along length  $L(T_0)$**
- **Length deficit,  $L(T_s)-L(T_0)$ , not connected**



# Set-in Stress Model

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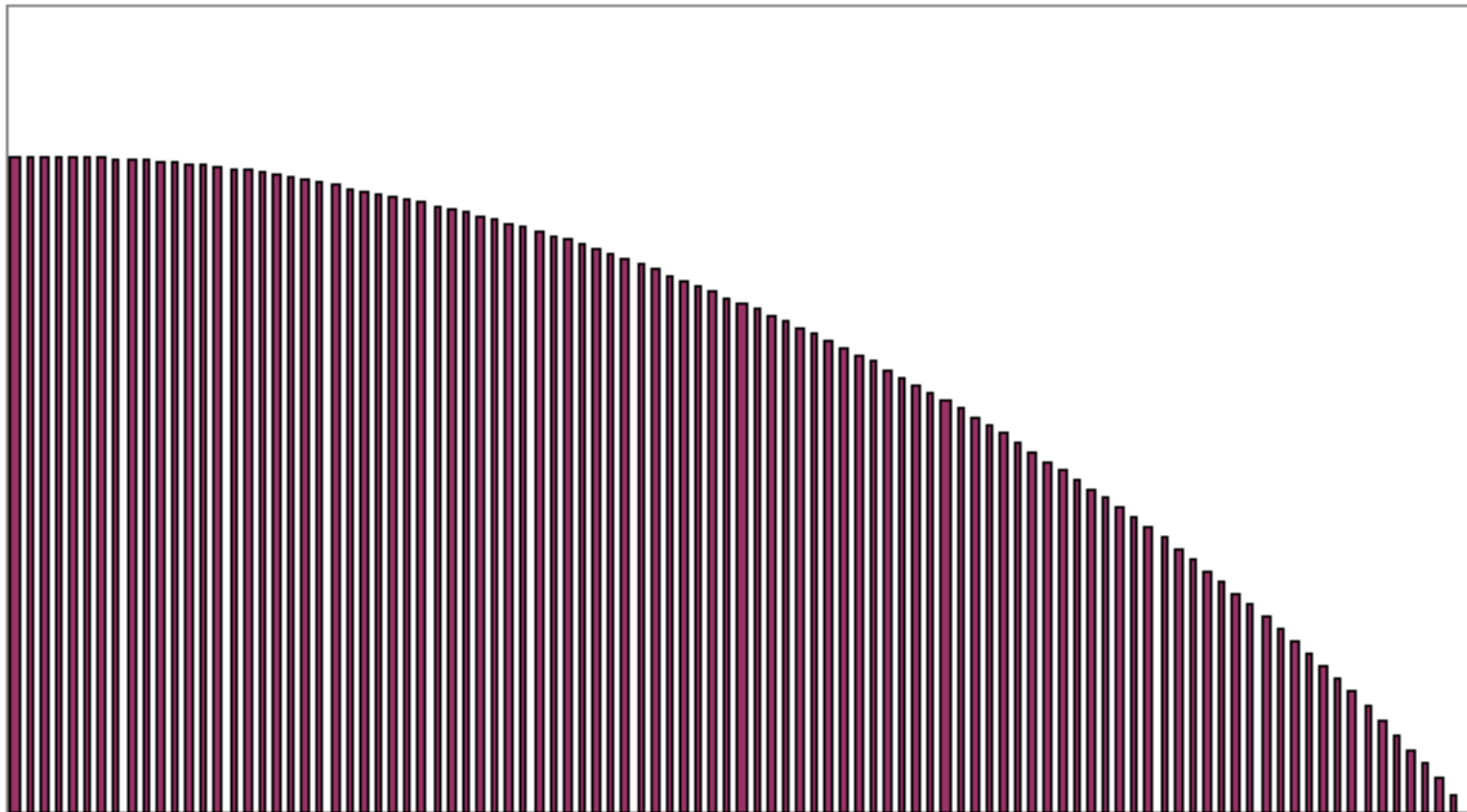
- **This process repeats as the annuli progressively solidify inward**
- **Length deficits result in dome shape on top**
- **Dome height larger with higher cooling rates**
- **Length deficit is the length not connected to the next outer annulus**
- **Length deficit causes set-in stress when the temperature becomes uniform**



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# Concentric Annuli Heights





# Total Length Deficit

- Total length deficit at any  $r$  is

$$L_d = \int_{R_0}^r \left. \frac{dL}{dr} \right|_{s^+} dr$$

- Derivative is on outer side of solidification front,  $s^+$



# Temperature Deficit

## ■ Temperature deficit at annulus I

$$\Delta T_i = T_s - T_{i+1}$$

- where the annulus at  $r_i$  has just solidified,
- and  $T_{i+1}$  is the Temperature of the annulus at  $r_{i+1}$  at that time.



# Total Temperature Deficit

- Total temperature deficit at any  $r$  is

$$T_d = \int_{R_o}^r \frac{dT}{dr} \Big|_{s^+} dr$$

- Derivative is on outer side of solidification front,  $s^+$
- Temperature deficit related to the length deficit by coefficient of thermal expansion



# SET-IN Stress (Pre-stressed)

- **Temperature Deficit Stress at Midplane at any  $r$**

$$\sigma_{\Delta i} = -\frac{E\alpha}{1-\mu} (\Delta T_{avg} - \Delta T_i)$$

- **Note the negative sign. Because deficit is the negative of the change in connective length.**



## Experiment Used to Verify

- **CWF2 is a prototype vertical ceramic waste form 1-m tall, 0.5-m diameter, 400-kg, encased in stainless steel can 0.5-cm thick.**
- **75% zeolite particles, 25% glass frit mixture  
Argon atmosphere furnace 10 day run**
- **Heat to melting (600 C), coalesces by over 50% to 925 C, holding at 925 C,**
- **Then cooling to solidification (600 C), then to room temperature.**



# Measurements Available

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- **Stress can not be measured directly during or after formation**
- **CWF encased in can in furnace, so cracks cannot be observed.**
- **Cracking times can be recorded – correlate with temperature and calculated stress.**
- **Destructive examination can determine where tensile cracking occurred.**



# Evidence of Cracking

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- **This cylinder developed many cracks during the low T portion of cooldown.**
- **At least 15 loud cracks were recorded over a period of 4 days. Sounds like gun shots.**
- **At the end of cooldown when the temperatures were below 400 C.**
- **Tensile Cracking pattern consistent with solidification stress theory**



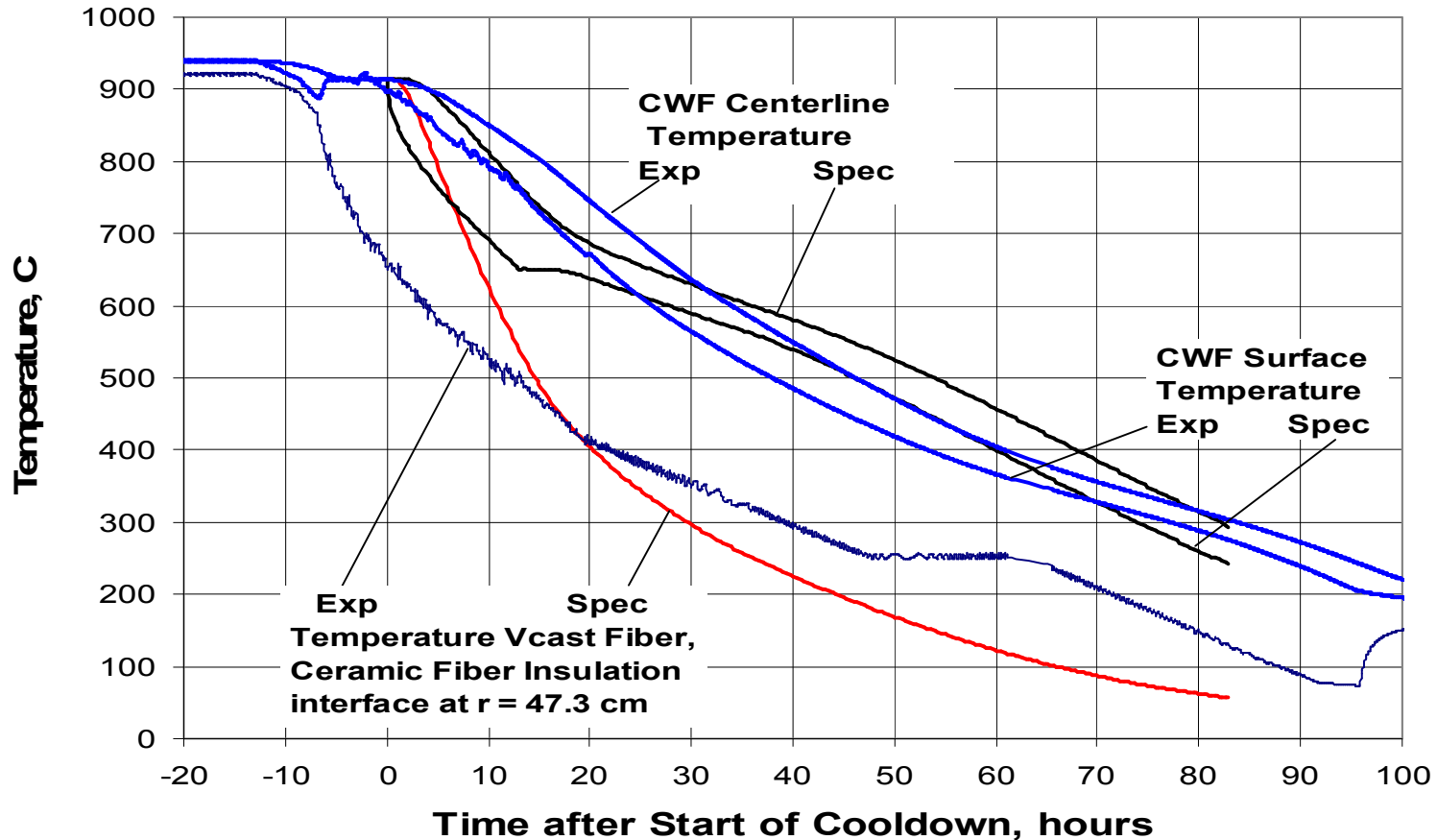
# Correlation of Cracking Sounds

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- **Surface and centerline temperatures at mid plane measured – allows calculation of midplane stress.**
- **Crack timing compared to calculated stress.**
- **Calculated total tensile stress exceeded the stress limit**
- **Verified cause of cracking is solidification stress, not thermal stress.**

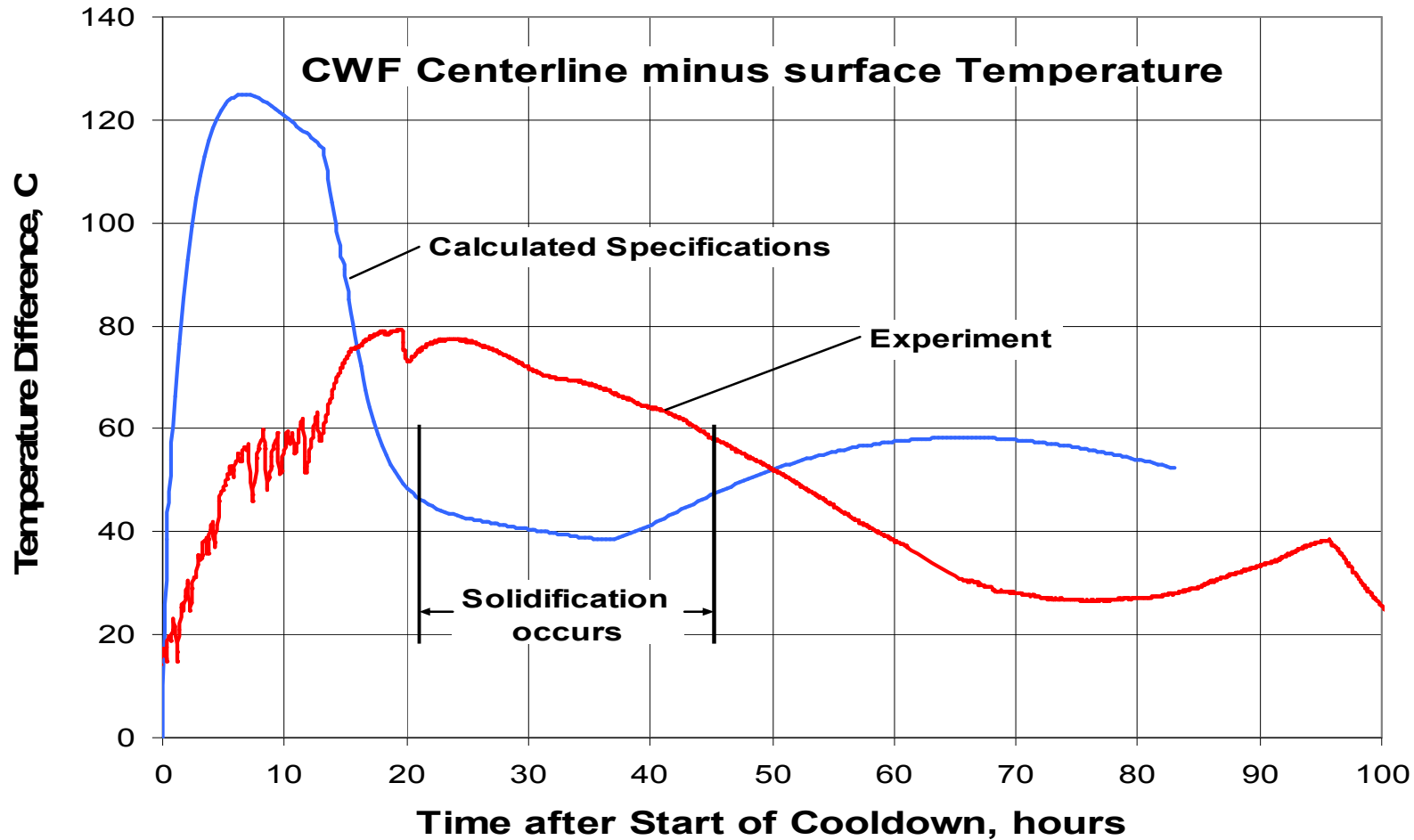


# Experimental Temperatures





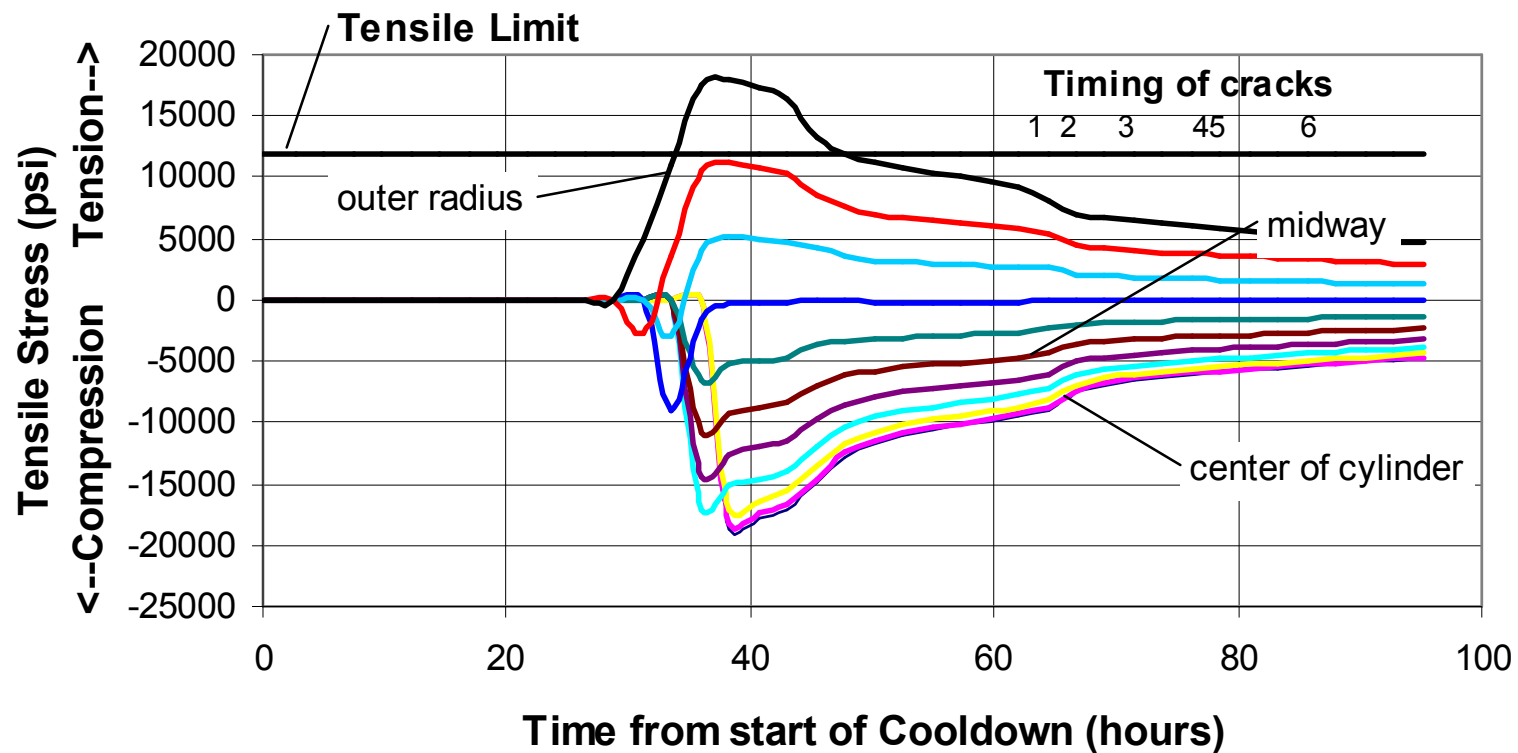
# Desired vs Actual Temperature Difference





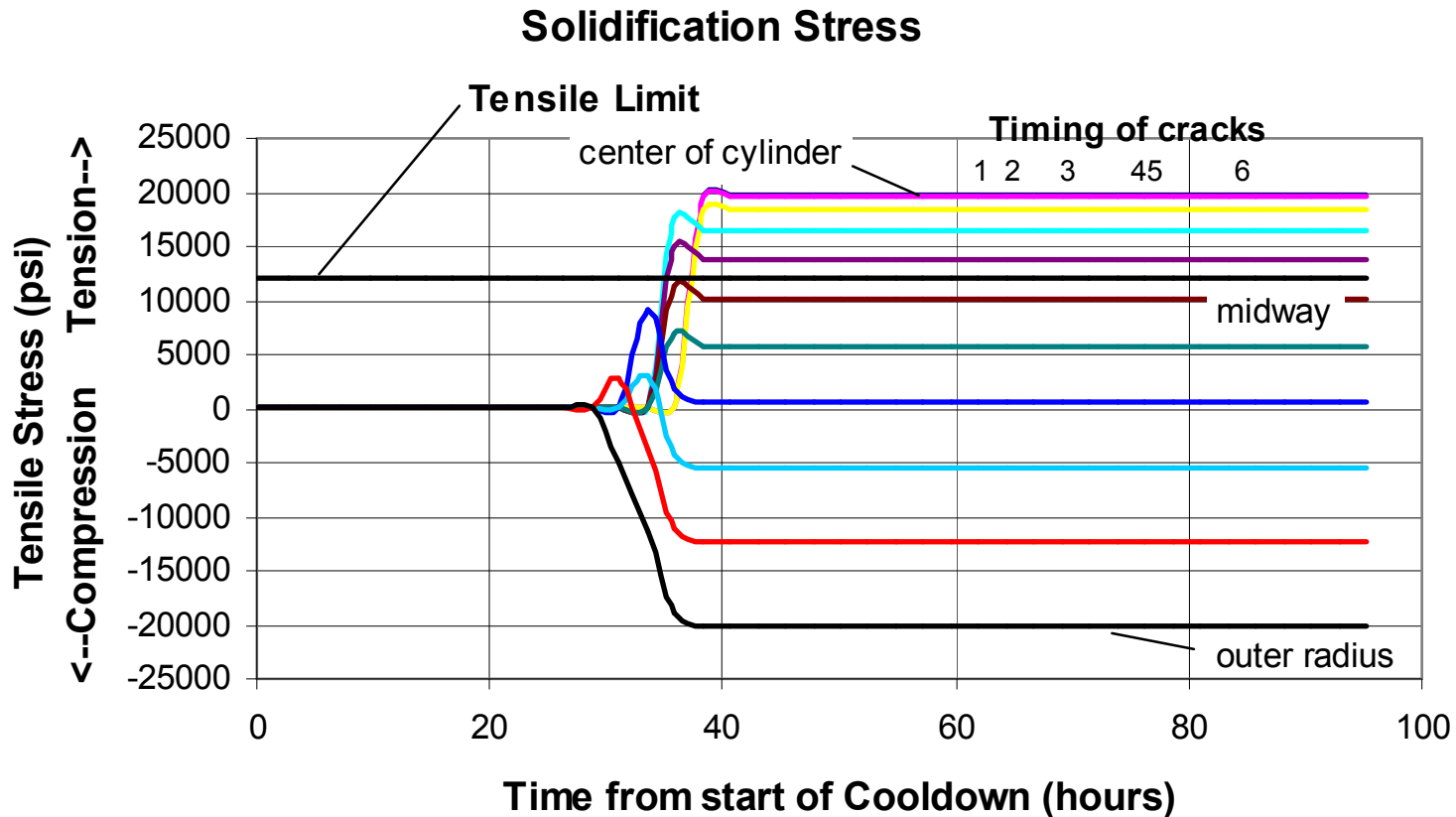
# Thermal Stress Below Limit

Thermal Stress



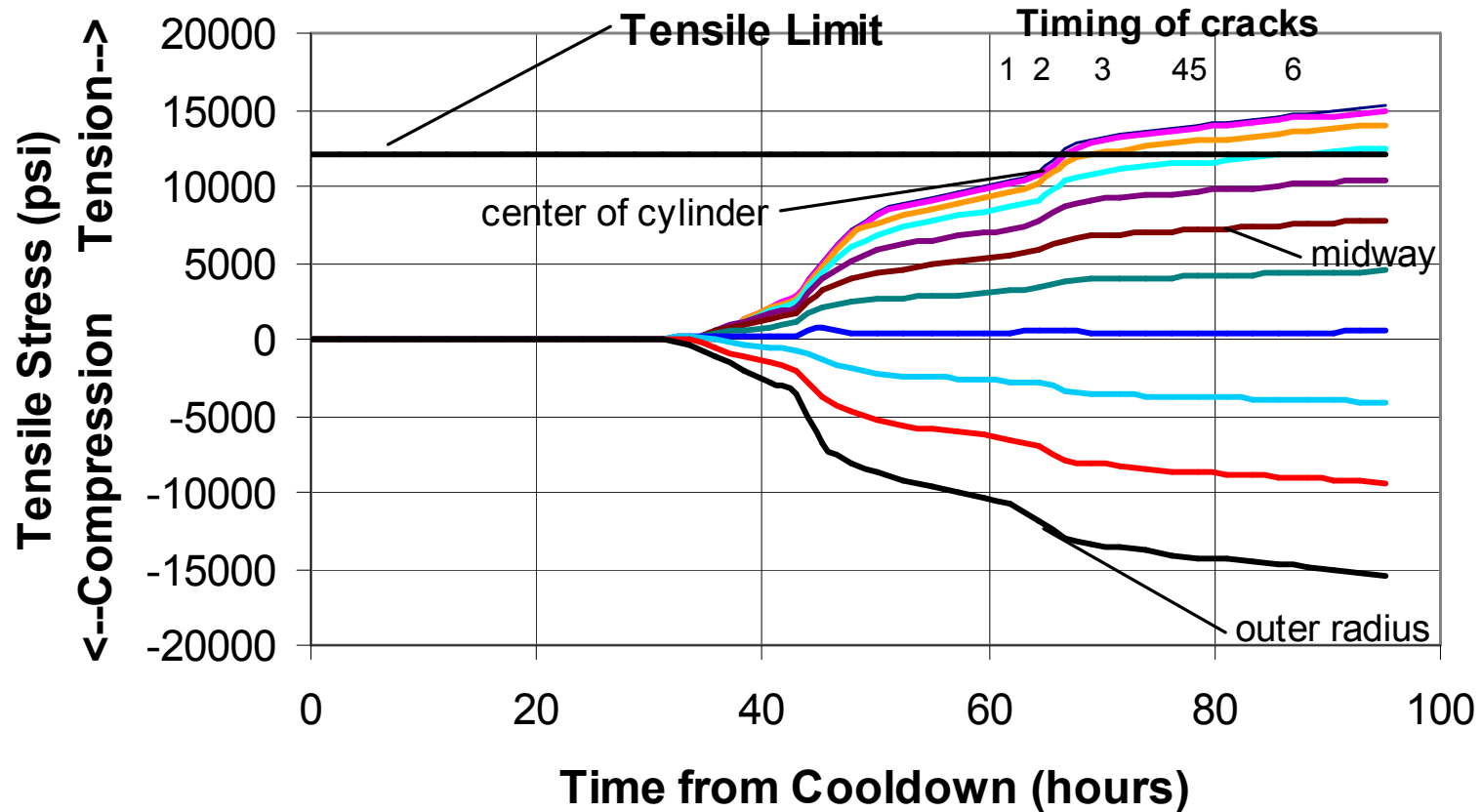


# Solidification Stress Exceeds Limit





# Total Stress Exceeds Limit

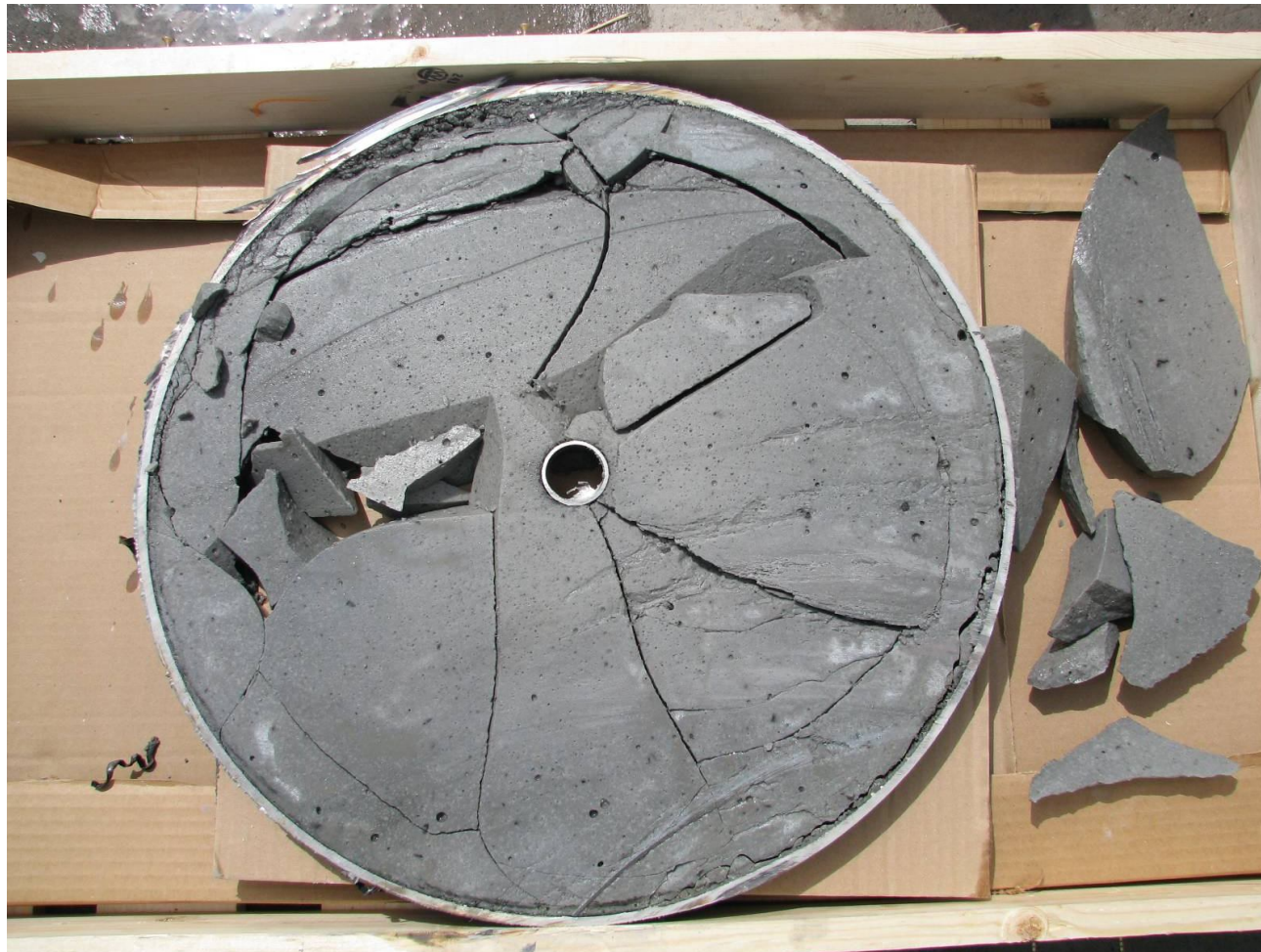




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# Ceramic Waste Form Damage





# Conclusions

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- **Cracking sounds determine when cracking occurs**
- **Destructive exam showed extensive cracking**
- **Cracking occurred during low temperature in inner region.**
- **Thermal stress alone predicted cracking at high temperatures in outer region.**
- **Solidification plus thermal stress predicts cracking at low temperatures in inner region.**
- **Experimental data from CWF2 verifies theory**